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ADVANTAGES AND LIMITATIONS OF ADAPTIVE AND NON-ADAPTIVE FORCE-BASED PUSHOVER PROCEDURES

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The recent drive for use of performance-based methodologies in design and assessment of structures in seismic areas has significantly increased the demand for the development of reliable nonlinear inelastic static pushover analysis tools. As a result, the recent years have witnessed the introduction of the so-called adaptive pushover methods, which, unlike their conventional pushover counterparts, feature the ability to account for the effect that higher modes of vibration and progressive stiffness degradation might have on the distribution of seismic storey forces. In this paper, the accuracy of these force-based adaptive pushover methods in predicting the horizontal capacity of reinforced concrete buildings is explored, through comparison with results from a large number of nonlinear time-history dynamic analyses. It is concluded that, despite its apparent conceptual superiority, current force-based adaptive pushover features a relatively minor advantage over its traditional non-adaptive equivalent, particularly in what concerns the estimation of deformation patterns of buildings, which are poorly predicted by both types of analysis.

Keywords: Force-based adaptive pushover; incremental dynamic analysis; FAP.

1. Introduction

Under the pressure of recent developments, seismic codes have begun to explicitly require the identification of sources of inelasticity in structural response, together with the quantification of their energy absorption capacity. Ideally, such performance evaluation of structural systems subjected to earthquake loading should be based on nonlinear time history analysis. However, the intrinsic complexity and the additional computational effort required by the latter (especially if a fibre-based distributed inelasticity modelling philosophy is adopted) do not justify its use in ordinary engineering applications.

As a result of the above, nonlinear static, as opposed to dynamic, pushover analysis has been gaining significance over recent years as a tool for assessment and design verification. Indeed, and despite its relative simplicity and ease of use, this numerical tool can provide information on many important response characteristics that cannot be obtained from an elastic static or dynamic analysis. Conventional pushover consists in the application and monotonic increase of a predefined lateral storey force pattern, kept constant throughout the analysis. However, such a procedure exhibits a number of limitations, mainly related to its inability to account for the progressive stiffness degradation, the change of modal characteristics and the period elongation of a structure subjected to monotonic loading. As a result, recent years have witnessed the introduction of the so-called adaptive pushover methods, which overcome, at least from a conceptual viewpoint, such limitations.

In this paper, and following a brief review of latest developments in the field, the accuracy of force-based adaptive pushover methods in predicting the horizontal capacity of reinforced concrete buildings is explored, through comparison with results from a large number of nonlinear time-history dynamic analyses. An extended version of the fully adaptive pushover algorithm proposed by Elnashai [2001], which manages to encompass all advanced features that have been recently proposed and developed by a number of researchers and can thus be deemed as a satisfactory representative of the current state-of-the-art in the field, is employed.

The results of the extensive parametric study carried out, summarised in the body of the current presentation, indicate that, despite its apparent conceptual superiority, current force-based adaptive pushover features a relatively minor advantage over its traditional non-adaptive counterpart, particularly in what concerns the estimation of deformation patterns of buildings, which are poorly predicted by both types of analysis. Therefore, possible areas of improvement of the adaptive scheme are also identified and suggested as potential solutions to the seemingly inherent shortcomings of current force-based pushover analysis methods.

2. Nonlinear Static Pushover Analysis in Earthquake Engineering

Pushover analysis has served well as an efficient and easy-to-use alternative to dynamic time-history analysis, since, despite its simplicity, it is capable of providing important structural response information. Indeed, pushover can be employed to identify critical regions, where inelastic deformations are expected to be high, and strength irregularities in plan or elevation that might cause important changes in the inelastic dynamic response characteristics [e.g. Krawinkler and Seneviratna, 1998]. In addition, pushover analysis can also provide realistic estimations of force demands in potentially brittle elements, such as shear-dominated members, or the consequences of strength deterioration of given members on the overall structural stability. Finally, this type of analysis is also capable of predicting the sequence of yielding and/or failure of structural components and the progress of the overall capacity curve of the structure, thus verifying the adequacy of the seismic load path (accounting for both structural and non-structural elements of the system).

However, conventional pushover analysis exhibits also shortcomings and limitations that confine its range of application and raise doubts about its effectiveness to accurately estimate structural seismic demand, as demonstrated by a number of researchers [Lawson *et al.*, 1994; Krawinkler and Seneviratna, 1998; Kim and D'Amore, 1999; Naeim and Lobo, 1999]. For instance, the deformation predictions can be highly inaccurate if higher modes are important and/or if the structure is pushed highly into its nonlinear post-yield range. The latter is characterised by gradual degradation and softening of the structural system during the analysis procedure, which in turn leads to significant elongation of the periods (see Fig. 1(a)) and change of modal shape characteristics (see Fig. 1(b)), as a result of the tendency for deformations to concentrate at the locations that sustain more damage.

Moreover, being a static method, pushover analysis reproduces material straining only, neglecting other sources of energy dissipation that are associated with dynamic response, such as kinetic energy and viscous damping, as well as duration effects. Finally, three-dimensional effects are difficult to incorporate, whereas the effects of cyclic earthquake loading cannot be modelled. In summary, pushover analysis lacks many important features of dynamic nonlinear analysis and thus cannot really substitute the latter in the role of the most accurate tool for structural analysis and assessment.

However, and nonetheless, several possible developments can considerably improve the efficiency of the method, bringing it some steps closer to the realistic modelling of nonlinear time-history analysis. Indeed, as shown further ahead, some of the aforementioned limitations can be overcome with the derivation of a fully adaptive procedure, which accounts for both higher mode contributions as well as alteration of the local resistance and modal characteristics of the structure, as induced by the progressive accumulation of damage. In this way, the stiffness degra-

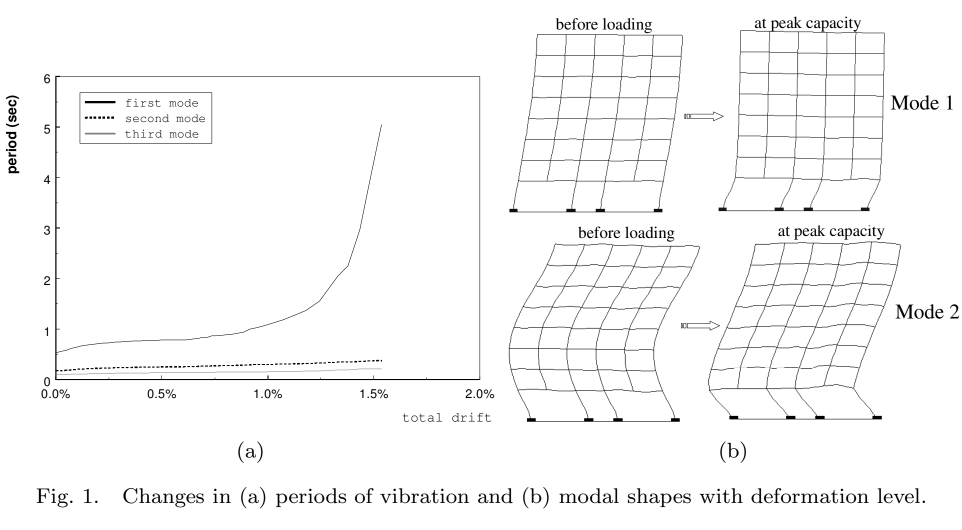


Fig. 1. Changes in (a) periods of vibration and (b) modal shapes with deformation level.

dation, the period elongation and the influence of all significant modes of vibration can be explicitly considered. In addition, finding a way to somehow incorporate the expected ground motion in the analysis might also allow the attainment of site-specific results, as required in some applications. This can be achieved with the utilisation of response spectra representative of such areas, derived from actual accelerograms or code provisions.

In recent years, a number of research endeavours have been carried out with the objective of introducing the aforementioned developments. The majority of such are briefly summarised in the subsections that follow.

2.1. *Pushover procedures considering higher-mode effects*

One of the first attempts to consider higher mode effects was made by Paret *et al.* [1996] and Sasaki *et al.* [1998], who suggested the simple, yet efficient, multi-modal pushover procedure (MMP). This comprises several pushover analyses under forcing vectors representing the various modes deemed to be excited in the dynamic response. The individual pushover curves are then converted to the Acceleration-Displacement Response Spectrum (ADRS) format, after which the Capacity Spectrum Method [Freeman, 1975] is utilised to compare the structural capacity with the earthquake demand. In this manner, it becomes evident which mode is more critical and where damage is likely to occur. The procedure is intuitive and does indeed provide qualitative information on higher mode effects, which conventional single mode pushover analysis fails to highlight. However, the effects of these higher modes cannot be easily quantified, since the method does not provide an estimation of the response.

A refinement of the multi-modal pushover procedure is the PRC method (Pushover Results Combination), which has been recently proposed by Moghadam and Tso [2002]. According to this method, the maximum seismic response is again estimated by combining the results of several pushover analyses, which are carried out using load patterns that match the modal shapes of a predefined number of vibration modes. The final structural response is obtained as a weighed (using the respective modal participation factors) summation of the pushover results from each analysis. Usually, the first 3 or 4 modes are considered.

A similar procedure, also based on MMP, is the MPA (Modal Pushover Analysis) method suggested by Chopra and Goel [2002]. According to MPA, the pushover curves corresponding to forcing vectors representing the various modes of vibration are idealised and transformed into bilinear curves of single-degree-of-freedom (SDOF) equivalent systems, so as to calculate the target deformation and the corresponding response parameters for each mode separately. The total demand is then determined by combining the peak modal demands using the SRSS rule. Typically, two or three modes are enough to achieve accurate results. When compared to Response Spectrum Analysis, the method gave rather good estimates for response parameters such as interstorey drifts or floor displacements, and the results presented were superior to the predictions of the pushover analyses with the fixed force distributions suggested by FEMA [ATC, 1997], which greatly underestimated drift demands.

Overall, the above multi-modal procedures constitute a significant improvement of conventional pushover analysis. The former are theoretically more robust and conceptually more attractive, since they explicitly consider the response of more than one vibration mode and the influence of the expected ground motion, thus yielding results that are closer to rigorous inelastic time-history analysis. However, these methods do not account for damage accumulation and resulting modification of the modal parameters, which might considerably affect the response characteristics of a given structure. Therefore, the fully adaptive methods presented in the following subsection seem to constitute a better alternative, due to their ability to adapt the load applied to the structure at different deformation levels (i.e. at different analysis steps), leading to conceptually more correct results.

2.2. *Adaptive pushover procedures*

Bracci *et al.* [1997] were the first to introduce a procedure that utilises fully adaptive patterns. The analysis starts by assuming an initial lateral load distribution, usually triangular, whereas the additional loads imposed in subsequent increments are calculated from the instantaneous base shear and storey resistances of the previous load step. The procedure was implemented in the dynamic analysis package IDARC [Valles *et al.*, 1996] leading to the attainment of apparently promising results. Lefort [2000] implemented an extended version of this method, which employed an additional force scaling equation to account for higher mode contributions, obtaining, however, limitedly-accurate response predictions.

A different adaptive methodology was proposed by Gupta and Kunnath [2000], in which the applied load is constantly updated, depending on the instantaneous dynamic characteristics of the structure, and a site-specific spectrum can be used to define the loading pattern. According to the method, eigenvalue analysis is carried out before each load increment, accounting for the current structural stiffness state. The number of modes of interest that are considered is predefined and the storey forces for each mode are estimated as the product between the modal participation factor, mass-normalised mode shape, weight of the storey and spectral amplification of the mode being considered. Then, a static analysis is carried out for each mode independently and the calculated action effects for each mode are combined with SRSS and added to the corresponding values from the previous step. At the end of the step, the structural stiffness state is assessed so as to be used in the eigenvalue analysis of the next step. The estimates of interstorey drifts and the sequence of the formation of local or global collapse mechanisms presented in the paper were satisfactory. However, the employment of SRSS to combine the different pushover responses, derived for each mode, implies that structural equilibrium is not satisfied at the end of each step.

Another method proposed by Requena and Ayala [2000], who discussed two variations of adaptive pushover (referred to as approaches 2-A and 2-B) and compared them with a modal fixed-pattern scheme. Whilst in one of the proposed variants the storey loads were derived through SRSS combination of modal forces, in the second option an "equivalent fundamental mode" is first determined (through an SRSS combination of the vibration modes shapes) and then used to compute the lateral loads. This procedure is repeated whenever the structural stiffness changes, as new plastic hinges form and develop, hence the lateral load distribution reflects the current state of inelasticity. The two alternative adaptive methods are indeed appealing since they are theoretically rigorous and they explicitly account for higher modes and spectral contributions. In addition, the procedures suggested in the paper for the determination of the target displacement are also of particular interest, albeit beyond the scope of the present work. However, and unfortunately, the analytical results presented by Requena and Ayala [2000] were only limited and the accuracy of the procedures could not be effectively assessed or judged.

A wholly alternative adaptive pushover methodology has been recently proposed by Albanesi *et al.* [2002], who suggested an Adaptive Energy-based Pushover Analysis (AEPOA) whereby the imposed lateral force/displacement profiles at each step supposedly take into account not only the inertial properties of the structure but also the kinetic energy that the latter is expected to mobilise when subjected to earthquake loading. The results obtained, however, did not seem particularly exceptional, since, compared to conventional pushovers, the curves derived by the AEPOA method were very unstable and tended to stop at very low deformation levels (less than 1% in some cases), whereas the predicted values of the base shear strength of the building were unreliable. In addition, no clear procedure for applying and updating the lateral loads (forces or displacements) was described.

Finally, Elnashai [2001] proposed an adaptive pushover scheme that seemed to encompass, within a single-analysis pushover algorithm, all advanced features described above. This single-run procedure is fully adaptive and multi-modal and accounts for system degradation and period elongation by updating the force distribution at every step (or at predefined steps) of the analysis. The dynamic properties of the structure are determined by means of eigenvalue analyses that consider the instantaneous structural stiffness state, at each analysis step. Site- or record-specific spectral shapes can also be explicitly considered in the scaling of forces, so as to account for the dynamic amplification that expected ground motion might have on the different vibration modes of the structure. This force-based adaptive pushover algorithm, deemed as a satisfactory representative of the current state-of-the-art in the field, was further developed and tested by the authors of the present work, for improved stability and accuracy, and is thus presented and discussed in greater detail in the remaining parts of this presentation.

3. The Force-Based Adaptive Pushover Algorithm (FAP)

The force-based adaptive pushover algorithm adopted and developed within the current presentation has been implemented in SeismoStruct [SeismoSoft, 2004], a fibre-modelling Finite Element program for seismic analysis of framed structures, which can be freely downloaded from the Internet. Full details on this computer package can be found in its accompanying manual. The implementation of the proposed algorithm can be structured in four main stages; (i) definition of nominal load vector and inertia mass, (ii) computation of load factor, (iii) calculation of normalised scaling vector and (iv) update of loading force vector. Whilst the first step is carried out only once, at the start of the analysis, its three remaining counterparts are repeated at every equilibrium stage of the nonlinear static analysis procedure, as described in the following subsections.

3.1. *Nominal load vector and inertia mass*

In conventional pushover, a nominal load vector P_0 is defined at the start of the analysis. The magnitude of such vector is of reduced relevance since it does not affect the attained results, due to the fact that pushover solution algorithms automatically scale the load vector in order to meet the analysis target. Its real usefulness resides instead on the fact that (i) it defines the structural nodes where the loads are applied and (ii) it characterises the load distribution shape (triangular, uniform, etc.) that is to be used throughout the entirety of the analysis.

In adaptive pushover, however, the loading vector shape is automatically defined and updated by the solution algorithm at each analysis step, for which reason the nominal vector P_0 must always feature a uniform (rectangular) distribution shape, in height, so as not to distort the load vector configuration determined in correspondence to the dynamic response characteristics of the structure at each analysis step (see Sec. 3.3). In other words, nominal loads, as defined at the start of the analysis, must be equal at all storeys, whilst their magnitude may be arbitrarily chosen (it is usually convenient to define an initial load vector with such magnitude so that the analysis load factor λ can be made to range between zero and unity, the latter corresponding to the attainment of full horizontal structural capacity, or thereabouts).

In addition, and still within the realms of the definition of the start-up conditions for the analysis, it is noteworthy that adaptive pushover requires the inertia mass M of the structure to be modelled, so that eigenvalue analysis, employed in the updating of the load vector shape (see Sec. 3.3), may be carried out. In the proposed adaptive pushover algorithm, both lumped and distributed mass elements may be employed, and freely spread throughout the structure.

The above highlights that, from a usability point-of-view, the implemented adaptive pushover algorithm effectively presents no additional effort and/or requirements with respect to its conventional non-adaptive counterpart, since the only element of novelty, in terms of analysis input, is the introduction of the building's inertia mass, which, however, can usually be obtained directly from the vertical gravity loads, already included in any type of pushover analysis.

3.2. *Computation of load factor*

In the proposed algorithm, the magnitude of the loading vector P at any given analysis step is obtained, in general terms, by the product of its nominal counterpart P_0, defined above, and the load factor λ at that step (see Eq. (1)). The latter is automatically increased, by means of a load control or response control incrementation strategy, until a predefined analysis target, or numerical failure, is reached.

$$P = \lambda \cdot P_0 \,. \tag{1}$$

Load control refers to the case where the load factor is directly incremented/controlled by the pushover algorithm whilst the structural response (e.g. nodal displacements and rotations) corresponding to such loading level (force level in this case) is subsequently determined. In other words, and in the case of FAP, a force-controlled pushover is carried out, with the load factor being used to scale directly the applied force vector until the point of peak capacity.

In the case of *response control*, on the other hand, it is the response of the structure (e.g. a given nodal displacement or rotation) that is first directly incremented/controlled, after which the load factor corresponding to such deformation level can be computed. The load factor variation, therefore, is not prescribed by the user, but is instead automatically calculated by the program so that the applied loading vector (force vector in this case) at a particular increment corresponds to the attainment of the target response displacement/rotation at the controlled node. It is noted that this type of load factor incremental scheme differs significantly from a displacement-controlled analysis since it is the response deformation of the structure, as opposed to a loading displacement vector, that is controlled by the program.

A more detailed description of the workings of such loading/solution schemes is beyond the scope of the current presentation, interested readers being instead referred to the manual of the FE program, indicated earlier, where the proposed adaptive pushover algorithm has been implemented. Here, it suffices to note that for the purpose of the current parametric study, and in general for all force-based adaptive pushover applications, the response control algorithm scheme is the preferred option since it (i) fully captures irregular response features such as soft-storey failures, (ii) models the softening post-peak loading/response branch of the structure, (iii) provides an even distribution of force-displacement curve points throughout both pre- and post-peak loading/response ranges and (iv) provides direct control of structural deformations, which, as widely acknowledged, provide a better insight into the damage incurred by the structure.

3.3. *Calculation of normalised scaling vector*

The normalised modal scaling vector, $\bar{F}$, used to determine the shape of the load vector (or load increment vector) at each step, is computed at the start of each load increment. In order for such scaling vector to reflect the actual stiffness state of the structure, as computed at the end of the previous load increment, an eigenvalue analysis is firstly carried out. To this end, the Lanczos algorithm [Hughes, 1987] is employed to determine the modal shapes and participations factors of any given predefined number of modes, after which the modal storey forces F_{ij} can be determined as:

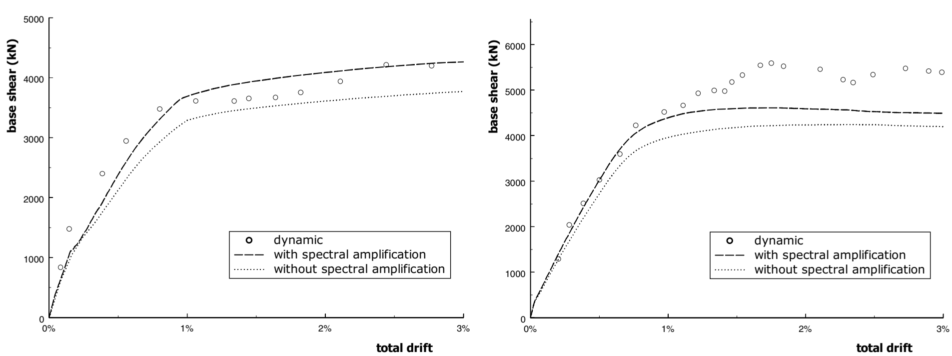
$$F_{ij} = \Gamma_j \phi_{ij} M_i \,, \tag{2a}$$

where i is the storey number and j is the mode number, Γ_j is the modal participation factor for the jth mode, ϕ_{ij} is the mass normalised mode shape value for the ith storey and the jth mode, and M_i is the mass of the ith storey.

Alternatively, the modal storey forces can be computed using Eq. (2b), given below, where an additional parameter $S_{a,j}$ that represents the acceleration response spectrum ordinate corresponding to the period of vibration of the jth mode, hereafter referred to as *spectral amplification*, is entered into the computation of F_{ij}. In other words, the modal storey forces are weighted by the S_a value at the instantaneous period of that mode, so as to take into account the effects that the frequency content of a particular input time-history, or spectrum, have in the response of the structure being analysed:

$$F_{ij} = \Gamma_j \phi_{ij} M_i S_{a,j} \,. \tag{2b}$$

Previous research [Mwafy and Elnashai, 2000; Antoniou, 2001] indicated that considering the spectral amplification of each particular mode in the computation of the F_{ij}, that is, the use of Eq. (2b) in favour of Eq. (2a), could contribute to an improvement in the similitude between static pushover and dynamic inelastic analysis



(a) Model WM15-AR (b) Model RM15-AR

Fig. 2. Effect of spectral amplification in adaptive pushover results.

results. These observations were confirmed in the present study, where the inclusion of ground motion characteristics in the adaptive pushover analysis (through the use of response spectra) provided an equally closer fit to the dynamic results, as shown in Fig. 2. Although in several cases the predictions with and without spectral amplification coincided, in none did the pushover with spectral amplification (i.e. different S_a for each mode) perform worse. Therefore, the utilisation of a spectral shape, either code-defined or derived from actual records, to determine modal storey forces is, in general, recommended.

Ideally, in order to assure full consistency between demand and supply ductilities, multiple response spectra, derived for varying values of equivalent viscous damping, should be employed so as to reflect the actual energy dissipation characteristics of the structure at each deformation level (i.e. at each analysis step). The implementation of such refinement, however, was beyond the scope of the current work, for which reason a single constant response spectrum derived for an equivalent viscous damping value of 5%, which the authors acknowledge may contribute to an overestimation of the first mode contribution, was used throughout the entirety of each analysis. Future work will target the assessment of the effects that such apparent limitation has on attained results.

The lateral load profiles of each vibration mode are then combined using either the Square Root of the Sum of Squares (SRSS, Eq. 3(a)), if the modes can be assumed as fully uncoupled, or the Complete Quadratic Combination (CQC, Eq. 3(b)) method, if cross-coupling of modes and respective viscous damping is to be considered. It is noted that in Eq. (3b), n stands for the number of modes whilst ρ_{jk} is the cross-model correlation coefficient, which can be approximated by Eq. (4) if one assumes all modes to feature the same damping coefficient ξ:

$$F_i = \sqrt{\sum_{j=1}^{n} F_{ij}^2}\,, \tag{3a}$$

$$F_i = \sqrt{\sum_{j=1}^{n}\sum_{k=1}^{n}(F_{ij}\cdot\rho_{jk}\cdot F_{ik})}\,, \tag{3b}$$

$$\rho_{jk} = \frac{8\cdot\xi^2\cdot(1+r)\cdot r^{1.5}}{(1-r^2)^2+4\cdot\xi^2\cdot r\cdot(1+r)^2}\,, \quad r = \frac{\omega_k}{\omega_j}\,. \tag{4}$$

Further, since only the relative values of storey forces (F_i) are of interest in the determination of the normalised modal scaling vector $\bar{F}$, which defines the shape, not the magnitude, of the load or load increment vector (see Sec. 3.4), the forces obtained by Eqs. (3a) or (3b) are normalised with respect to the total value, as follows:

$$\bar{F}_i = \frac{F_i}{\Sigma F_i}\,. \tag{5}$$

It is noted that, as highlighted by Priestley [2003], the use of SRSS or CQC rules to combine modal forces may not always lead to the attainment of an adequate load vector shape that accurately represents the dynamic response characteristics of the structure at a particular deformation level. In fact, if true reproduction of dynamic analysis response is the objective of the pushover analysis, then the normalised scaling vector should ideally be obtained through a weighted vectorial addition of the contribution of each mode, to avoid the not necessarily realistic storey force increase when the force vector from one mode is summed, through SRSS or CQC, to the fundamental one. The employment of such alternative modal force combination procedure, however, calls for additional in-depth studies that are conspicuously beyond the scope of the present endeavour. Hence, and taking also account of the fact that in the present work observed modes of vibration at every deformation stage were always sufficiently apart, the SRSS modal combination procedure was consistently employed throughout the analyses.

It is equally noteworthy that when the structural response reaches its post-peak range, the eigen-solver can no longer output real eigen-solutions, due to the presence of negative values in the diagonal of the stiffness matrix which in turn lead to imaginary periods of vibration, corresponding to wholly unfeasible modal shapes. In such cases, the load vector shape is no longer changed (only its magnitude is updated), effectively meaning that a conventional non-adaptive pushover analysis is employed thereafter. This is acceptable since, in the majority of applications, entrance in the post-peak response range corresponds to the formation of a given failure mechanism, which then remains qualitatively unchanged until collapse is reached. Furthermore, and as described in Sec. 4.3, when a given storey starts failing due to damage accumulation, force-based adaptive algorithms tend to generate unrealistically high load concentrations at that given storey, leading to equally unfeasible high drift values. Hence, and for this reason also, the employment of a constant load vector shape in the post-peak softening structural response range seems justified.

Finally, in cases where a very large number of analysis steps are employed and/or when the structural model is very large, it might prove advantageous for the load vector shape to be updated with a frequency lower than the number of analysis steps, so as to reduce the computation effort. Such feature is explicitly available in the algorithm developed by the authors. Within the framework of the current parametric study, however, for reasons of accuracy and analysis stability, the load vector shape was updated at every analysis step, with up to ten modes of vibration being considered in its computation.

3.4. *Update of loading force vector*

Once the normalised scaling vector and load factor or load factor increment have been determined, and knowing also the value of the initial nominal load vector, the loading force vector at a given analysis step can be updated using one of two alternatives; total or incremental updating.

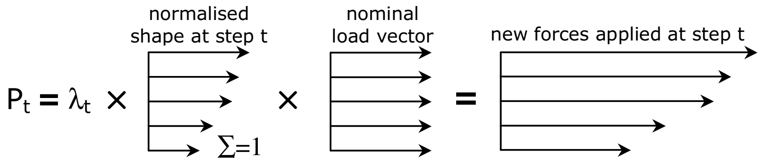


Fig. 3. Graphical representation of loading force vector calculation with total updating.

3.4.1. *Total Updating*

With *total updating*, the load vector P_t at a given analysis step t is obtained through a full substitution of the existing balanced loads (load vector at previous step) by a newly derived load vector, computed as the product between the current total load factor λ_t, the current normalised modal scaling vector $\bar{F}_t$ and the nominal load vector P_0, as schematically represented in Fig. 3 and numerically translated into Eq. (6a):

$$P_t = \lambda_t \cdot \bar{F}_t \cdot P_0 \,. \tag{6a}$$

This load updating scheme does seem to be the most adequate if one is looking to reproduce in full the dynamic response characteristics of a structure subjected to earthquake loading, since the load vector is fully updated at every single analysis step to reflect the stiffness state, hence the vibration properties, of the structure at that particular deformation level. This effectively means that an adaptive pushover that has been started with a, say, first mode of vibration triangular force distribution, may then be concluded with an applied force distribution that reflects a second or third mode of vibration, which might be controlling the response of the structure at that stage of the analysis. This behaviour was clearly observed in some of the case studies considered in the parametric study described in Sec. 4, as schematically illustrated in Fig. 4 below.

Unfortunately, in addition to its auspicious full reproduction of inertia force distribution patterns observed in nonlinear dynamic analysis, Total Updating features also a conspicuous lack of numerical stability, due to the difficulty in introducing the abrupt type of storey force changes depicted in Fig. 4 below. This is particularly evident in those cases where the applied force at one storey level reduces with an increasing load factor, whereas the forces at the adjacent levels augment. In his work, Antoniou [2002] does present a solution to this numerical obstacle; if at one or more storeys there is reduction of the applied forces, all storey loads are

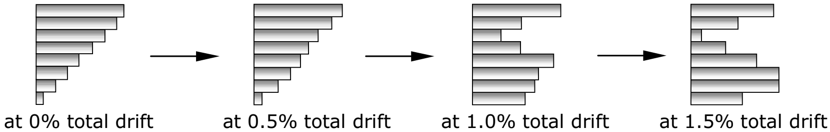


Fig. 4. Total-Updated loading force vector at different deformation levels (model WH30-NR1).

proportionally increased so that at any given storey the loading force is at least equal to that of the previous step. This loading vector corrective technique, however, cannot, for obvious reasons, be employed with a response control algorithm, the preferred choice for force-based adaptive pushover (Sec. 3.2, above). Therefore, Total Updating has not been adopted in the current parametric study, being instead replaced by the stable, and comparably accurate, Incremental Updating scheme, described below.

3.4.2. *Incremental Updating*

With *Incremental Updating*, the load vector P_t at a given analysis step t is obtained by adding to the load vector of the previous step P_{t-1} (existing balanced loads) a newly derived load vector increment, computed as the product between the current load factor increment $\Delta\lambda_t$, the current modal scaling vector $\bar{F}_t$ and the nominal load vector P_0, as schematically represented in Fig. 5 and numerically translated into Eq. (6b).

$$P_t = P_{t-1} + \Delta\lambda_t \cdot \bar{F}_t \cdot P_0 \,. \tag{6b}$$

It is noteworthy that the results obtained with Incremental Updating were consistently very similar to those obtained with Total Updating, in those cases where the latter could indeed be applied. This seems to come as a result of the fact that in both schemes the trends of the load distributions are similar, as shown by Antoniou [2001, 2002], even though the absolute values of the forces differ significantly in some cases. In addition, even in those limited cases where differences between the two load updating modalities were not negligible, it was by no means clear which variant provided better estimates. Finally, it is also noted that Bracci *et al.* [1997], Gupta and Kunnath [2000] and Requena and Ayala [2000], whose work has been briefly reviewed in Sec. 2 of this paper, have also adopted the use of incremental load vector updating.

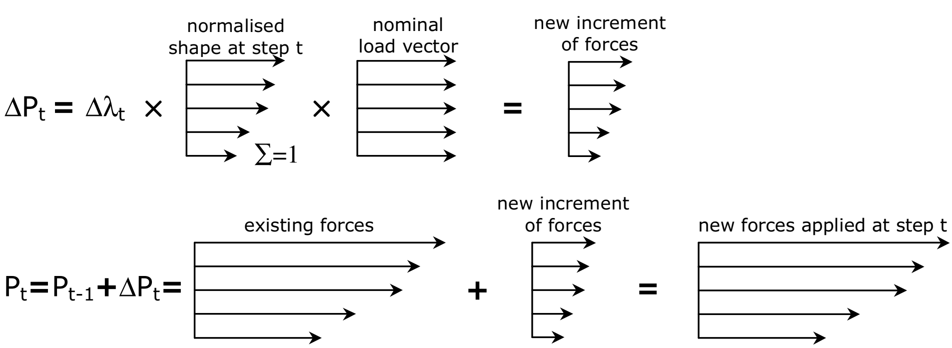


Fig. 5. Graphical representation of loading force vector calculation with incremental updating.

4. Parametric Study

In what follows, sample results of an extensive parametric study that aimed at assessing the effectiveness of the proposed force-based adaptive pushover algorithm (FAP), are presented. To this end, results obtained with the latter are compared to well-established static and dynamic analysis methods, all of which have been applied to a large number of case studies, as discussed below.

4.1. *Introduction*

Adaptive pushover is intended to be a method for general use in design and assessment. It is therefore imperative to verify its efficiency for different structural configurations subjected to equally diverse input ground motions. Extensive preliminary testing, comprising 1200 dynamic and pushover analysis on 12 different "stick-models", has already been carried out by Antoniou [2001], for which reason the current study focused instead in the application of the proposed algorithm to more realistic structural models. A series of frame systems were thus considered and analysed, using conventional pushover procedures as well as the suggested adaptive analysis scheme. Furthermore, and in order to fully assess the method's accuracy, or lack of, a considerable number of dynamic analyses were conducted, with the corresponding results being then compared with those from force-based adaptive pushover.

Three different structural configurations were employed in this study; a 12-storey regular frame, an eight-storey irregular frame and a dual (wall-frame) system. Moreover, different ductility classes and design ground accelerations were considered, resulting in a total of 12 structural models. The latter represent common reinforced concrete structures and are based on buildings designed and detailed at the University of Patras [Fardis, 1994], seemingly according to the 1995 version of

Table 1. Definition of the structural systems considered.

Group	Structure Reference	Structural System	No. of Storeys	Ductility Level	Design PGA (g)	Behaviour Factor (q)	$T_{\text{uncracked}}$ (sec)
1	RF-H030/RH30	Regular Frame	12	High	0.30	5.00	0.697
	RF-M030/RM30			Medium		3.75	0.719
	RF-M015/RM15			Medium	0.15	3.75	0.745
	RF-L015/RL15			Low		2.50	0.740
2	IF-H030/IH30	Irregular Frame	8	High	0.30	4.00	0.565
	IF-M030/IM30			Medium		3.00	0.536
	IF-M015/IM15			Medium	0.15	3.00	0.613
	IF-L015/IL15			Low		2.00	0.614
3	FW-H030/WH30	Regular Frame-Wall	8	High	0.30	3.50	0.569
	FW-M030/WM30			Medium		2.625	0.557
	FW-M015/WM15			Medium	0.15	2.625	0.601
	FW-L015/WL15			Low		1.75	0.588

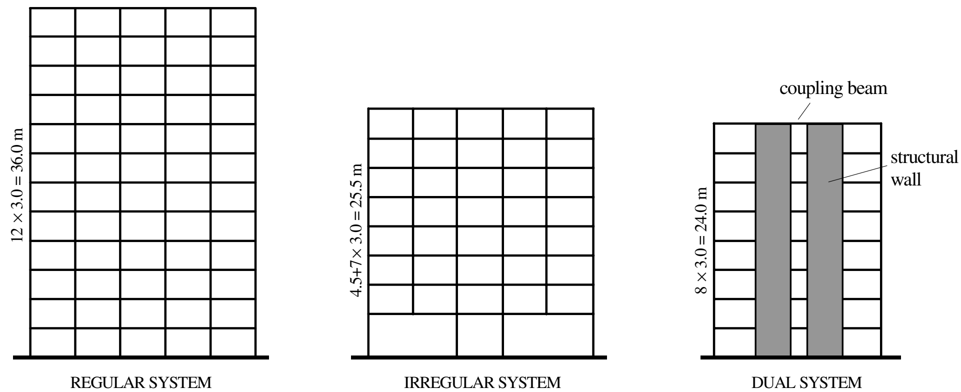


Fig. 6. Geometric characteristics of the regular, irregular and dual systems.

Eurocode 8 [CEN, 1995]. Subsequently, they were modelled by Mwafy [2001] under the framework of a different project, and were then adapted by Rovithakis [2001] for the purpose of the current research endeavour. Their general characteristics are defined in Table 1 and schematically illustrated in Fig. 6.

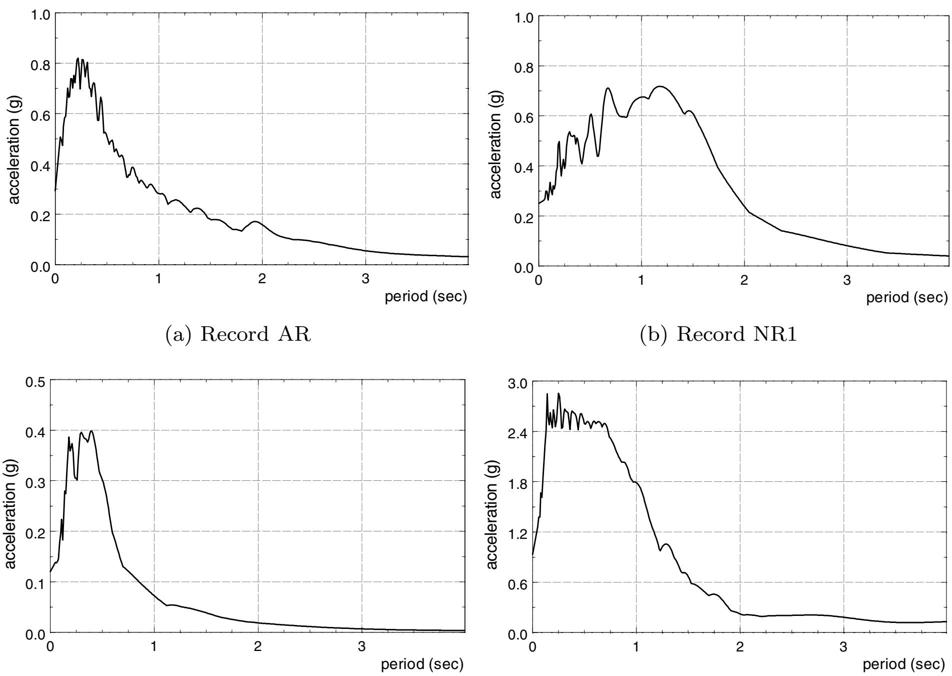
It is also worth noting that many of the modelled buildings feature a column-sway, rather than beam-sway, type of failure mechanism, and are thus likely to experience significant modifications of the applied force vector shape at different deformation levels. This serves well the purpose of the current study, since it is in such non-beam-sway cases that conventional non-adaptive pushovers struggle the most to accurately reproduce the dynamic response characteristics of a building subjected to a given input motion.

Four input time-histories, consisting of one artificially-generated accelerogram [Campos-Costa and Pinto, 1999] and three natural records (Loma Prieta earthquake, USA, 1989), were employed for the dynamic analyses of the study. The selection of these four records aimed at guaranteeing that the twelve buildings described above would be subjected to a wide-ranging type of earthquake action, in terms of frequency content, peak ground acceleration, duration and number of high amplitude cycles. Indeed, the original PGA of the records varies between $0.12g$ and $0.93g$, the spectral shapes are markedly distinct and provide high amplifications at different periods (see Fig. 7), and the ratio between the significant duration (defined as the interval between the build up of 5% and 95% of the total Arias Intensity [Bommer and Martinez-Pereira, 1999]) and the total duration ranges from 22% to 72%. The characteristics of the records are summarised in Table 2, whereas their elastic response spectra for an equivalent viscous damping of 5% are shown in Fig. 7.

The twelve building configurations subjected to four records resulted in a total number of $12 \times 4 = 48$ test cases, deemed adequate to provide a representative sample for the verification of the proposed algorithm. Two conventional pushover procedures were carried out for each of the 12 buildings, for which the two distributions

Table 2. Characteristics of the records employed in this study.

Record	Peak Ground Acceleration	Peak Response Acceleration	5% AI Threshold	95% AI Threshold	Total Duration t_{tot}	Significant Duration t_{eff}	t_{eff}/t_{total}
AR	$0.30g$	$1.28g$	2.32 sec	12.75 sec	15.0 sec	10.43 sec	69.5%
NR1	$0.25g$	$0.90g$	11.23 sec	20.16 sec	40.0 sec	8.93 sec	22.3%
NR2	$0.12g$	$0.50g$	1.02 sec	9.52 sec	33.2 sec	8.50 sec	25.6%
NR3	$0.93g$	$4.25g$	1.44 sec	8.68 sec	10.0 sec	7.24 sec	72.4%



(a) Record AR

(b) Record NR1

(c) Record NR2

(d) Record NR3

Fig. 7. Elastic response spectra of the four records (5% equivalent viscous damping).

proposed in the NEHRP Guidelines [ATC, 1997] were employed; the uniform distribution, whereby lateral forces are proportional to the total mass at each floor level, and the triangular distribution, in which seismic forces are proportional to the product of floor mass and storey height. Furthermore, in excess of ten adaptive pushovers were run per building, employing the different variants of FAP discussed in Sec. 3. The results of the 144 pushover analyses carried out were then compared to the envelopes derived with the recently proposed Incremental Dynamic Analysis (IDA) procedure [Hamburger *et al.*, 2000; Vamvatsikos and Cornell, 2002; Mwafy and Elnashai, 2000; Papanikolaou, 2000], whereby a structure is subjected to a series of nonlinear time-history analysis of increasing intensity (e.g. peak ground acceleration is incrementally scaled from a low elastic response value up to the attainment of a pre-defined post-yield target limit state). To this end, about 20 inelastic dynamic analyses with variable scaling factors were performed per case, leading to a total of approximately 1000 nonlinear time-history runs.

All the analyses, dynamic or static, were carried out up to the point of 3% global drift, employing the previously introduced FE package, capable of predicting the large displacement behaviour of space frames under static or dynamic loading, taking into account both local (beam-column effect) and global (large displacements/rotations effects) geometric nonlinearities as well as material inelasticity. The spread of the latter along the member length and across the section area is explicitly represented through the employment of a fibre modelling approach, implicit in the formulation of the inelastic beam-column frame elements employed in the analyses. Structural members were subdivided into 4–5 elements, with smaller elements at member ends so as to ensure that inelasticity could be accurately modelled. Beams and columns were modelled as extending from the centre of one beam-column joint to the centre of the next, in order to take account, albeit in an indirect and empirical manner, of joint flexibility that could be induced by joint shear distortion, yield penetration and/or bar slip. Inertia mass was taken as permanent vertical load plus 30% of its variable counterpart. No viscous damping was considered in the dynamic analysis, since energy dissipation through hysteresis is already implicitly included within the nonlinear fibre model formulation of the inelastic frame elements, and non-hysteretic type of damping was assumed to be negligible within the scope of the present application.

In order to appraise the applicability and the effectiveness of force-based adaptive pushover, a series of top displacement versus base shear plots (capacity curves) has been created, comparing the adaptive and conventional pushover results against the dynamic analysis envelopes obtained for each of the four accelerograms described above. The interstorey drift and storey shear profiles, more representative of local response characteristics, at four different deformation levels (0.5%, 1.0%, 1.5% and 2.5% total drift) have also been plotted. Typically, the 0.5% plots described the elastic (or better, pre-yield) behaviour of the buildings, those of 1.0–1.5% signalled the point of global yielding, where the stiffness changes significantly and the load distributions are rapidly updated, whilst 2.5% global drift is deep within the inelastic range.

Finally, it is noted that by comparing pushover results with IDA output obtained for each single accelerogram, as opposed to the statistical average of all dynamic cases, a much more demanding and precise assessment of the static procedures is effectively carried out, since structural response peculiarities introduced by individual input motions are not smoothed out through results averaging. Within this non-statistical verification framework, and in order to facilitate interpretation of the most important observations and exemplification of the significant conclusions, only representative plots are given henceforth.

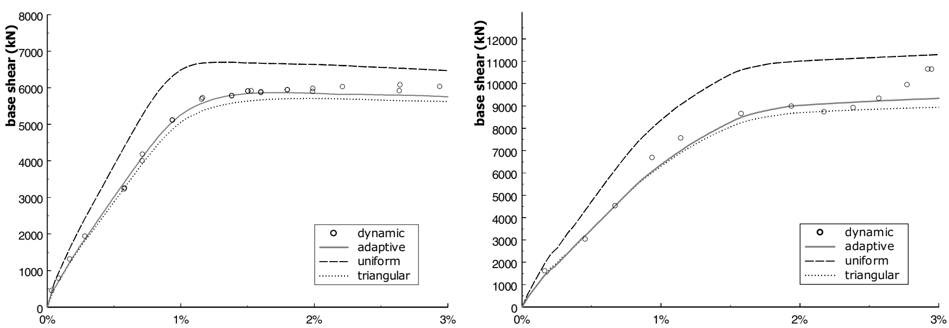
4.2. *Capacity curves*

As noted in Sec. 3, the most advantageous force-based pushover variant, when both numerical stability and results accuracy are considered, is that featuring response control, spectral amplification and incremental updating. Consequently, in all plots provided hereafter, only such FAP configuration will be compared with conventional pushover (uniform and triangular distributions), as well as IDA results.

It is also noted that in the current work the dynamic analysis envelopes consist of the locus of maximum total drift versus corresponding base shear (i.e. peak base shear within a time-window (±0.5 seconds of the instant of maximum drift occurrence). This reflects a pragmatic, rather than judicious, choice, since it has not been possible to unequivocally demonstrate that any of the three available choices for plotting dynamic analyses envelopes (max drift versus max shear, max drift versus corresponding shear, corresponding drift versus max shear) is more meaningful than the other, for all structural types and analysis cases. Future work will hopefully clarify this matter, with the authors not excluding the possibility that the use of only one of the three possible dynamic analysis results interpretation to match an equally unique force-based adaptive pushover might be deemed as inappropriate.

In Fig. 8, representative cases of comparative plots for models with different structural characteristics, and for different records, are shown. It is observed that the IDA points lay between the uniform and the triangular distributions, at least within the pre-yield range (some noticeable exceptions where these two distributions did not provide upper and lower bounds, even at pre-yield range, have been recorded, as discussed later in this section). The adaptive pushover curves are typically located between the two conventional curves as well, providing a slightly closer fit to the IDA points.

In addition to the above, however, there have also been several examples where the pushover methods (adaptive or not) failed to correctly predict the dynamic



(a) Model RL15-NR1

(b) Model WM30-NR3

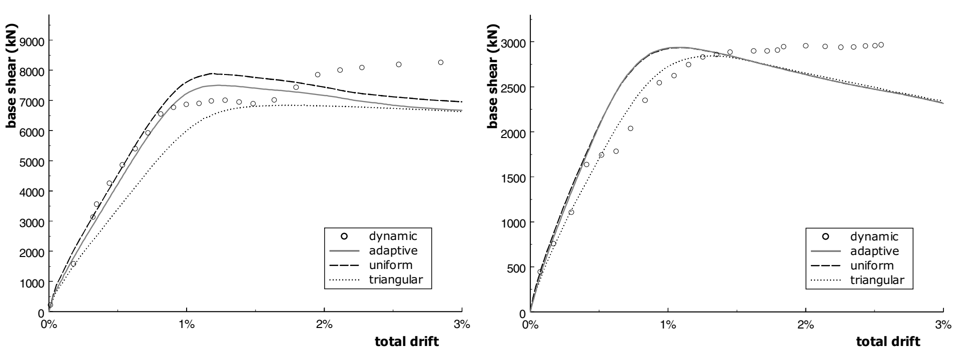
Fig. 8. Adaptive and conventional pushover curves versus dynamic analysis envelopes.

structural behaviour throughout the entire deformation range, even in the pre-yield region. The most serious complications have been caused by record NR2, which features an acceleration response spectrum with large amplifications for periods up to 0.4 sec followed by a very steep descending branch (see Fig. 7), which, associated to the softening of the building's response and consequential period elongation, lead to scenarios where higher modes dominate the structural behaviour of the studied structures. This resulted in highly irregular interstorey drift and storey shear profiles, shown in Sec. 4.3, and accordingly peculiar IDA envelopes. Consequently, none of the force-based pushover procedures employed here, including the adaptive algorithm, managed to capture the characteristics of the dynamic response of structural systems such as those illustrated in Fig. 9.

As noted in Fig. 9, in some cases, the dynamic base shear was considerably higher than the static shear estimates provided by the uniform distribution (or, less frequently, lower than the triangular). These are induced by the highly irregular shapes of the inertia forces at the time step of maximum displacement, which cannot be adequately reproduced, throughout the entire deformation range, by any of the force-based static procedures employed in this work. Moreover, on many of such cases, the slope of the IDA envelope was seen to change abruptly at a certain point of the curve (e.g. Fig. 9(a), drift of 1.7%), due to the fact that, for some of these structures, when subjected to a given accelerogram with a certain frequency content, the increase of scaling factors, thus deformation demand and hence periods of vibration, would at a certain point introduce a sudden change in the dominant response mode of vibration, which would then lead to a markedly diverse response trend, clearly identified in the dynamic analysis envelopes. Similar observations have been made by Vamvatsikos and Cornell [2002] and Antoniou [2001].

4.3. *Interstorey drifts and shear profiles*

Interstorey drifts are crucial parameters in terms of structural response, especially in view of the recent developments in the field of performance-based engineering,



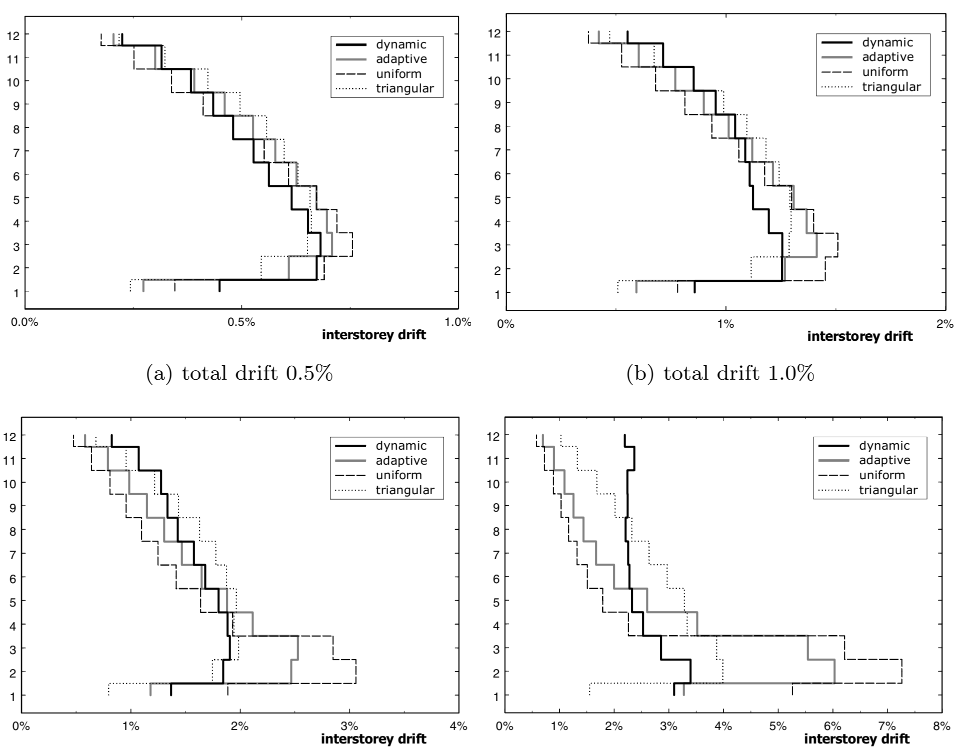
(a) Model RH30-NR2 (b) Model IM15-NR2

Fig. 9. Adaptive and conventional pushover curves versus dynamic analysis envelopes.

since they are closely related to the damage sustained by a building. Therefore, it is essential for static methods to provide not only correct capacity curves, but also drift values as close as possible to the predictions of more rigorous dynamic analyses.

For this reason, the drift profiles obtained in each of the pushover and dynamic analyses were examined at the four levels of total displacement introduced earlier; 0.5%, 1.0%, 1.5% and 2.5%. It is noted that the drift profiles from the dynamic analysis represent in fact envelopes of peak response at the specified total displacement levels, not actual profiles at a given instant of time. In other words, they represent the drift maxima at each storey as obtained in the one time-history analysis (20 were carried out for each pair of building/record scenarios) where their sum comes the closest to each of the aforementioned total displacement limits.

In Fig. 10, the sequence of the storey deformations for one representative case is shown. In general, the predictions were reasonably accurate in the pre-yield and reasonably early inelastic range for both the adaptive and conventional procedures (Figs. 10(a) and 10(b)). Although there was no clear "global winner", the force-based adaptive pushover predictions were found to be slightly more accurate in the majority of cases, in consistency with the observations on the capacity curves,



(a) total drift 0.5%

(b) total drift 1.0%

(c) total drift 1.5%

(d) total drift 2.5%

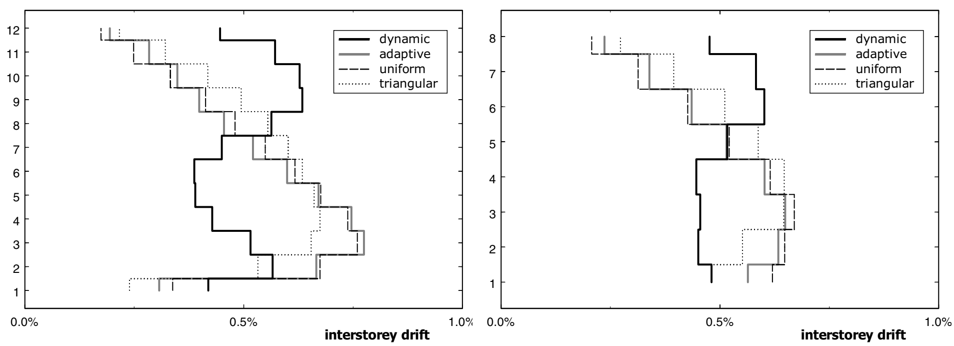
Fig. 10. Model RM30-NR3. Interstorey drift profiles at different deformation levels.

shown above. However, in the highly inelastic region, the FAP method failed to provide correct response estimates, by exaggerating the inelastic deformations in the locations of damage and underestimating them elsewhere (Figs. 10(c) and 10(d)). Such erroneous behaviour seems to be related to the fact that at onset of damage on a given storey, the resulting reduction in stiffness and consequent changes in the modal shapes derived by eigenvalue analysis at the corresponding analysis step induces a concentration at that storey of the increment of forces for the next step, thus increasing even more its damage and hence resulting in a sort of "vicious circle" that leads to exaggeratedly large deformation levels at that such damaged locations.

It is also noteworthy that, whereas the non-adaptive pushover with uniform distribution yielded equally flawed results in that range, its triangular counterpart, despite its fixed loading pattern, managed to provide better drift estimates at high deformation levels. This can be explained by the fact that, typically, the damage occurred at the lower floor levels for all structural configurations and, contrary to the uniform and the adaptive patterns, with the triangular distribution the forces applied to the failed storeys were smaller than those applied to the upper storeys, resulting in more reasonable drift predictions. However, and although the above does indeed constitute an interesting observation, it should be stressed that such good performance of the fixed triangular patterns comes as a result of the location of damage in the considered case studies. Had damage occurred at the upper floors, the triangular distribution would have failed to predict accurately the drifts profiles. Such behaviour was indeed observed by Antoniou [2002], where non-adaptive pushovers with triangular distributions failed to provide adequate predictions for those structural models that featured a soft-storey located at mid-height.

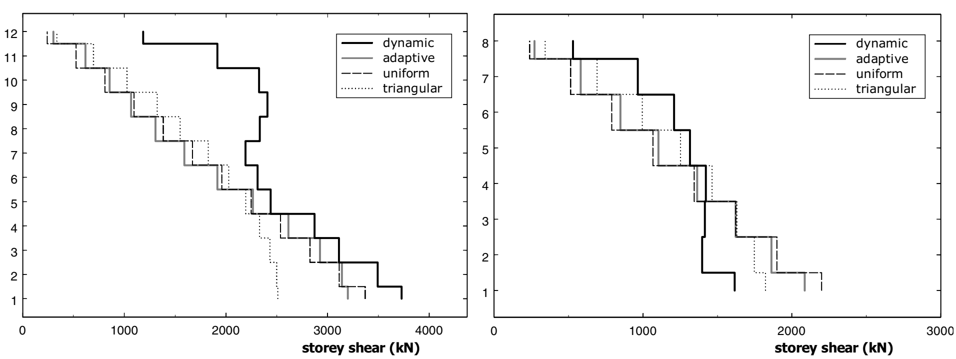
Similar observations were made also for the storey shear profiles obtained from each of the pushover and dynamic analyses at the aforementioned four levels of structural deformation (0.5%, 1.0%, 1.5% and 2.5% total drift). Although differences were not significant, FAP shear distribution predictions were found to be typically between the results from the uniform and triangular non-adaptive pushover analyses, and usually closer to the dynamic envelopes, at least for total deformations levels of up to 1.5%. Thereafter, it was conventional pushover with triangular force distribution that presented the best predictions, for the reasons discussed above.

There have also been cases where the interstorey drift and shear distribution predictions of the adaptive and the fixed distributions were totally inadequate, even at the early stages of deformation. Typical examples of such flawed behaviour are depicted in the drift profiles of Fig. 11 and shear distributions of Fig. 12, the latter clearly confirming that for a full reproduction of dynamic response characteristics it is required that static analysis methods feature the capability of reversing the sign of applied storey forces (in Fig. 12(a), for instance, dynamic inertia forces applied at storeys eight and nine are negative). This is a feature that, as highlighted in Sec. 3.3, current force-based pushover schemes, adaptive or not, seem to be lacking.



(a) Model RL15-NR2 (b) Model IH30-NR2

Fig. 11. Interstorey drift profiles at 0.5% total drift for different model configurations.



(a) Model RL15-NR2 (b) Model IL15-AR

Fig. 12. Storey shear profiles at 0.5% total drift for different model configurations.

It is interesting to note that the above deficiencies were not always reflected on the capacity pushover curves, discussed previously, which, on occasions, were seen to be correct even for those cases where storey drift and/or shear profiles were incorrect. This highlights the importance of considering local parameters, such as storey drift and/or storey shear profiles, when assessing the efficiency of pushover methods, adaptive or not.

5. Conclusions

In this work, the accuracy of adaptive and non-adaptive force-based pushover methods in predicting the seismic response characteristics of reinforced concrete buildings has been explored. To this end, a fully adaptive pushover algorithm, which explicitly accounts for the effects that progressive stiffness degradation and higher-mode effects have on structural response, has been developed and employed in the verification study. The latter comprised comparisons between the capacity curves and drift/shear profiles obtained through static and dynamic nonlinear fibre-based analysis on a number of real structural models of different configurations and ductility classes, subjected to equally diverse input ground motions; each pushover was compared with the IDA output obtained for each individual accelerogram, as opposed to the average of the latter, to avoid smoothening out of important structural response peculiarities introduced by individual records.

A summary of the main observations and general conclusions of the present study is presented below:

- Effective assessment of the accuracy of pushover algorithms, through comparison with time-history analysis results, should not be based on the comparison of capacity curves alone, but rather include an evaluation and comparison of local response quantities, such as storey drift and storey shear profiles.
- Non-adaptive pushover analyses, with the triangular and uniform distributions, do not always provide curves that constitute a lower and an upper bound to the IDA response points. Moreover, they fail to reproduce accurately the local dynamic response characteristics of buildings, particularly within the post-peak range.
- Force-based adaptive pushover provides slightly improved capacity curves over its conventional counterparts, with a closer fit to the IDA points. Although the differences were not always impressive, they were clear in the vast majority of the cases.
- Accordingly, up to global yield, FAP derives interstorey drift and storey shear profiles closer to the dynamic envelopes. However, in the highly inelastic range it tends to introduce excessive forces at the locations of plastic hinges and, subsequently, to overestimate the drift values at such storeys.
- In some of the studied cases, none of the force-based static analysis methods employed in this study could correctly predict the dynamic response characteristics of buildings, even at pre-yield deformation stages, due to the difficulties faced by current force-based pushover algorithms in modelling a reversal of applied storey force sign.
- Overall, it can be stated that force-based adaptive pushover, in its present state of development, features a relatively minor advantage over its traditional non-adaptive counterparts, all of which present limitations in the accurate prediction of dynamic response characteristics of RC buildings.

The above clearly indicates that further research work is required before force-based adaptive pushover can be considered as a valid alternative to nonlinear time-history analysis. As highlighted in the body of the document, such developments might include (i) the implementation of multiple response spectra to determine the spectral amplification of different modes at varying deformation stages and/or (ii) the employment of weighted vectorial addition, as opposed to SRSS and CQC combination rules, in the computation of normalised scaling vector, as suggested by Priestley [2003].

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References

Albanesi, T., Biondi, S. and Petrangeli, M. [2002] "Pushover analysis: An energy based approach," *Proc. of the Twelfth European Conference on Earthquake Engineering*, London, United Kingdom, Paper No. 605.

Antoniou, S. [2001] "Pushover analysis for seismic assessment of RC structures," *Transfer Report*, Department of Civil Engineering, Imperial College London, United Kingdom.

Antoniou, S. [2002] "Advanced inelastic static analysis for seismic assessment of structures," *PhD Thesis*, Imperial College London, University of London, United Kingdom.

ATC [1997] "NEHRP Guidelines for the seismic rehabilitation of buildings," *FEMA Report No. 273*, Federal Emergency Management Agency, Applied Technology Council, Washington, D.C.

Bommer, J. and Martinez-Pereira, A. [1999] "The effective duration of earthquake strong motion," *Journal of Earthquake Engineering* **3**(2), 127–172.

Bracci, J. M., Kunnath, S. K. and Reinhorn, A. M. [1997] "Seismic performance and retrofit evaluation of reinforced concrete structures," *Journal of Structural Engineering* **123**(1), 3–10.

Campos-Costa, A. and Pinto, A. V. [1999] "European seismic hazard scenarios — An approach to the definition of input motion for testing and reliability assessment of Civil Engineering structures," *JRC Special Publication No. X.99.XX*, Joint Research Centre, Ispra, Italy.

CEN [1995] European Prestandard ENV 1998-1-1: Eurocode 8 — Earthquake resistant design of structures, Part 1: General rules and rules for buildings. *Comite Europeen de Normalisation*, Brussels, Belgium.

Chopra, A. K. and Goel, R. K. [2002] "A modal pushover analysis procedure for estimating seismic demands for buildings," *Earthquake Engineering and Structural Dynamics* **31**, 561–582.

Elnashai, A. S. [2001] "Advanced inelastic static (pushover) analysis for earthquake applications," *Structural Engineering and Mechanics* **12**(1), 51–69.

Fardis, M. N. [1994] "Analysis and design of reinforced concrete buildings according to Eurocodes 2 and 8. Configurations 3,5 and 6," *Reports on Prenormative Research in Support of Eurocode 8.*

Freeman, S. A., Nicoletti, J. P. and Tyrell, J. V. [1975] "Evaluations of existing buildings for seismic risk — A case study of Puget Sound Naval Shipyard, Bremerton, Washington," *Proc. of the First U.S. National Conference on Earthquake Engineering*, Oakland, California, pp. 113–122.

Gupta, B. and Kunnath, S. K. [2000] "Adaptive spectra-based pushover procedure for seismic evaluation of structures," *Earthquake Spectra* **16**(2), 367–391.

Hamburger, R. O., Foutch, D. A. and Cornell, C. A. [2000] "Performance basis of guidelines for evaluation, upgrade and design of moment-resisting steel frames," *Proc. of the Twelfth World Conference on Earthquake Engineering*, Auckland, New Zealand, Paper No. 2543.

Hughes, T. J. R. [1987] *The Finite Element Method, Linear Static and Dynamic Finite Element Analysis* (Prentice-Hall, Inc.).

Kim, S. and D'Amore, E. [1999] "Push-over analysis procedure in earthquake engineering," *Earthquake Spectra* **15**(3), 417–434.

Krawinkler, H. and Seneviratna, G. D. P. K. [1998] "Pros and cons of a pushover analysis of seismic performance evaluation," *Engineering Structures* **20**(4–6), 452–464.

Lawson, R. S., Vance, V. and Krawinkler, H. [1994] "Nonlinear static push-over analysis — Why, when, and how?" *Proc. of the Fifth US National Conference on Earthquake Engineering*, Earthquake Engineering Research Inst., Oakland, California, **1**, 283–292.

Lefort, T. [2000] "Advanced pushover analysis of RC multi-storey buildings," *MSc Dissertation*, Department of Civil and Environmental Engineering, Imperial College London, United Kingdom.

Moghadam, A. S. and Tso, W. K. [2002] "A pushover procedure for tall buildings," *Proc. of the Twelfth European Conference on Earthquake Engineering*, London, United Kingdom, Paper No. 395.

Mwafy, A. M. and Elnashai, S. A. [2000] "Static pushover versus dynamic-to-collapse analysis of RC buildings," *ESEE Research Report No. 00/1*, Department of Civil and Environmental Engineering, Imperial College London, United Kingdom.

Mwafy, A. M. [2001] "Seismic performance of code-designed RC buildings," *PhD Thesis*, Imperial College London, University of London, United Kingdom.

Naeim, F. and Lobo, R. [1999] "Avoiding common pitfalls in pushover analysis," *Proc. of the Eighth Canadian Conference on Earthquake Engineering*, Canadian Association for Earthquake Engineering, Vancouver, British Columbia, Canada, pp. 269–274.

Papanikolaou, V. [2000] "Development and verification of adaptive pushover analysis procedures," *MSc Dissertation*, Department of Civil and Environmental Engineering, Imperial College London, United Kingdom.

Paret, T. F., Sasaki, K. K., Eilbeck, D. H. and Freeman, S. A. [1996] "Approximate inelastic procedures to identify failure mechanisms from higher mode effects," *Proc. of the Eleventh World Conference on Earthquake Engineering*, Disc 2, Paper No. 966.

Priestley, M. J. N. [2003] *Myths and Fallacies in Earthquake Engineering, Revisited* (IUSS Press, Pavia, Italy).

Requena, M. and Ayala, G. [2000] "Evaluation of a simplified method for the determination of the nonlinear seismic response of RC frames," *Proc. of the Twelfth World Conference on Earthquake Engineering*, Auckland, New Zealand, Paper No. 2109.

Rovithakis, A. [2001] "Verification of adaptive pushover analysis procedures," *MSc Dissertation*, Department of Civil and Environmental Engineering, Imperial College London, United Kingdom.

Sasaki, K. K., Freeman, S. A. and Paret, T. F. [1998] "Multi-mode pushover procedure (MMP) — A method to identify the effects of higher modes in a pushover analysis," *Proc. of the Sixth US National Conference on Earthquake Engineering*, Earthquake Engineering Research Inst., Oakland, California, 12 pp.

SeismoSoft [2004] *SeismoStruct — A Computer Program for Static and Dynamic Nonlinear Analysis of Framed Structures* [online]. Available from URL: http://www.seismosoft.com.

Valles, R. E., Reinhorn, A. M., Kunnath, S. K., Li, C. and Madan, A. [1996] *IDARC2D Version 4.0: A Computer Program for the Inelastic Analysis of Buildings*, *Technical Report NCEER 96-0010*, State University of New York at Buffalo, Buffalo, NY.
Vamvatsikos, D. and Cornell, C. A. [2002] "Incremental dynamic analysis," *Earthquake Engineering & Structural Dynamics* **31**(3), 491–514.